Abstract
The motor Circle Diagram (also known as a Heyland, Behrend, or Ossanna diagram) is simply a graphical representation of the locus of induction motor stator current from zero speed to synchronous speed. The concept of the Circle Diagram was conceived in the late 19th century, but is today no longer commonly utilized by engineers. With the advent of modern computational techniques utilizing the motor exact equivalent circuit, stator currents and torques can be calculated faster and with more accuracy than by graphical means. However, the Circle Diagram is invaluable in providing an intuitive understanding of motor characteristics throughout the entire range of operation.

Introduction
The Circle Diagram is a graphical solution of the motor equivalent circuit which traces the locus of stator current, $I_1$, at all values of slip, $0 \leq s \leq 1$, in the complex plane. The Circle Diagram however is not a phasor diagram, as a phasor diagram is the solution of an equivalent circuit for a single operating condition. As a motor will continuously change its operating condition as it accelerates from zero speed to full load speed, the equivalent circuit changes as well.

From this single figure (Figure 1), the Circle Diagram allows the engineer to graphically visualize all motor parameters, such as real power, reactive power, torque, current, efficiency, power factor, and losses, for all operating points.

Circle Diagram Required Information
As shown in Figure 2, the Circle Diagram is a semi-circle drawn in the complex plane, located in the first quadrant with the real and imaginary components plotted on the vertical and horizontal axes, respectively. As the diagram is semicircular, two points are necessary to trace the graph. The information required to construct the diagram is acquired from the no load test and the locked rotor test. The full load test can be used in place of the locked rotor, although considering the full load current is approximately six times less in magnitude, it may not yield as accurate a result.

Drafting of the Circle Diagram can be done manually or by a computer, although manual construction can provide a more intuitive understanding.
No-Load Test
At no load and rated voltage, motor losses are comprised of stator copper losses, stator core losses, and rotational losses. For the purposes of the Circle Diagram, the test yields the magnetizing branch current vector, $\bar{I}_m$, as shown in Figure 3, necessary to plot the first point, A.

As the motor has no load, the rotor will rotate nearly at synchronous speed, with the non-zero slip only being required to compensate for rotational losses.

Locked-Rotor Test
During the locked rotor condition, current flow is limited predominantly by the machine leakage inductance, which is much smaller than the magnetizing reactance. Due to this, locked rotor current can typically reach 6 or more times the normal operating current. For the purposes of the circle diagram, the locked rotor test yields the leakage branch current vector, $\bar{I}_2$, as shown in Figure 6, necessary to plot the second point, B.

Power input during the locked rotor condition is comprised mainly of (above 99%) stator and rotor $I^2R$ losses.

$A'A'' = \text{Stator Core and Copper Losses}$
Constructing the Circle Diagram
From these two points, A and B, the circle can now be drawn. As the stator core and copper losses are constant throughout the range of motor slip, the bottom of the circle is a horizontal line parallel to the imaginary axis.

From point A, a circle is drawn through $\pi/2$ radians, which satisfies the equation of the rotor referred stator current (1):

$$I_2 = \frac{V_1}{\sqrt{(R_1+R'_2/s)^2+(X_1+X'_2)^2}} \quad (1)$$

The impedance angle of the equivalent circuit can be described by (3):

$$\theta = \sin^{-1}\left(\frac{(X_1+X'_2)}{\sqrt{(R_1+R'_2/s)^2+(X_1+X'_2)^2}}\right) \quad (2)$$

Combining equations (1) and (2) yields the equation of the circle diagram (3), which when added vectorially to the no load current, traces the locus of stator current as a function of slip.

$$I_2 = \frac{V_1}{(X_1+X'_2)} \sin \theta \quad (3)$$

The diameter of the circle is a function of the total leakage reactance.

$$Diameter \approx \frac{V_1}{(X_1+X'_2)} \quad (4)$$

Note that the images in this paper have vastly accentuated angles and magnitudes in order to properly illustrate the components of the diagram. In practice, many of the losses shown would be very difficult to see and draw without the aid of a computer.

Geometric Observations from the Circle Diagram
The circle diagram is located in the first quadrant only for convenience, but could be projected in any direction. As the real axis is vectorially in the same direction as the applied voltage, any line segment drawn vertically represents total real power (kW), while any line segment drawn horizontally represents total reactive power (kvar).

Line Segments at Locked Rotor (B)
Consider an induction motor at rated voltage with a stationary rotor. The locked rotor current vector is drawn to point B, as shown in Figure 7, represents the mechanical power developed. This line segment is thus referred to as the mechanical power developed line. Since the rotor is stationary, there is no mechanical power output, and thus any input power must go towards real losses. The line segment BB'' must then be the total stator and rotor losses.

$$BB'' = Stator and Rotor Losses$$

The line segment BB’ can be split between the rotor and stator losses by the following equations:

$$BB' \approx \frac{I_{LR}^2 R_1}{V_1} \quad (5)$$

$$B'B'' \approx \frac{I_{LR}^2 R'_2}{V_1} \quad (6)$$

$$B'BB'' = Stator Copper Losses$$

Equations 5 and 6 draw an important conclusion, that being the ratio of the line segments BB’ and B’B” is the ratio of rotor to stator resistances.

$$\frac{BB'}{B'B''} \approx \frac{R'_2}{R_1} \quad (7)$$
As the amount of starting torque is directly a function of the amount of rotor copper losses at locked rotor, the length of the BB' segment may also represent the amount of starting torque at rated voltage. Since the amount of torque developed is dictated by the amount of real power transferred across the air gap, a line drawn from A' to B' is referred to as the torque line or air gap line.

**Maximum Developed Torque**

As torque is a function of real power, and real power is represented by vertical line segments, the maximum developed torque can be found at the point along the circle arc with the largest real line segment to the torque line. This can be found graphically by drawing a line from the center of the circle base to the circle arc that is normal to the torque line, as shown in Figure 9.

Similarly to maximum torque, the maximum output power obtained graphically is also not readily obtained experimentally.

**Line Segments at the Operating Point (C)**

The operating point is the steady state condition where motor output torque is equal to the rated load required torque. Once all the lines have been plotted, a graphical solution of the motor operating parameters can be found, as shown in Figure 11.

Note that the maximum torque obtained by graphical means is not readily obtained experimentally.

**Maximum Developed Power**

Referring back to the No-Load test, the difference in current from point A to point A' represents the motor rotational losses. Adding the rotational losses to the maximum developed power line yields total output. Thus, the output line is a line segment drawn from point A parallel to the maximum developed power line. The maximum output power is found at the point along the circle arc with the largest real line segment to the output line. This can be found graphically by drawing a line from the center of the circle base to the circle arc that is normal to the output line, as shown in Figure 10.

Note that slip is equivalent to the rotor copper losses divided by air gap power.

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**Symbols and Units**

- \( CC' = \) Total Input Power
- \( CC_0 = \) Total Output Power
- \( \frac{CC_0}{CC'} = \) Efficiency
- \( \frac{CC'}{CC_0} = \) Power Factor
- \( \frac{C_P C_T}{CC_T} = \) Slip (per-unit)
Assumptions and Error
The motor circle diagram constructed and analyzed in this paper uses a perfect circle. In practice however, this circle does not have a constant radius or base.

Motor locked rotor tests are done similarly to transformer percent impedance tests. That is, the equivalent leakage reactance is found by applying a voltage level such that rated stator current flows, and then is extrapolated to the rated voltage. When locked rotor current actually flows in the windings, iron core saturation will occur, reducing the leakage reactances. This will distort the circumference of the circle in the regions of high current flow.

Constructing the circle diagram makes use of the induction motor approximate equivalent circuit rather than the exact equivalent circuit. As the magnetizing branch is moved ahead of the stator impedance branch, this brings about error in the stator core and copper losses from the no-load test. As a result, the circle base is not actually parallel to the imaginary axis.

References

About the Author
Daniel Lang holds a Bachelor of Science in Electrical Engineering from the University of Alberta (2010). He has worked as an engineering consultant in all areas of power systems design and analysis, from small commercial buildings to large scale heavy industrial installations.

His work has included power system analyses and design with an emphasis on protective relaying, balanced and unbalanced fault analysis, steady-state / loadflow and transient analysis, arc flash analysis, and protection logic.

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